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Fatigue life prediction model for accelerated testing of electronic components under non-Gaussian random vibration excitations

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ABSTRACT

In this paper, a novel fatigue life prediction model for electronic components under non-Gaussian random vibration excitations is proposed based on random vibration and fatigue theory. This mathematical model comprehensively associates the vibration fatigue life of electronic components, the characteristics of vibration excitations (such as the root mean square, power spectral density, spectral bandwidth and kurtosis value) and the dynamic transfer characteristics of an electronic assembly (such as the natural frequency and damping ratio) together. Meanwhile a detailed solving method was also presented for determining the unknown parameters in the model. To verify the model, a series of random vibration fatigue accelerated tests were conducted. The results obtained show that the predicted fatigue life based on the model agreed with actual testing. This fatigue life prediction model can be used for the quantitative design of vibration fatigue accelerated testing, which can be applied to assess the long-term fatigue reliability of electronic components under Gaussian and non-Gaussian random vibration environments.

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1. Introduction

Random vibration environments have long been the cause of many fatigue induced failures of electronic components. To ensure electronics' robustness and reliability during their operation or transportation, timely validation of the long-term durability of electronic components under their service vibration environment is needed. It is usually done by laboratory vibration testing. However, operational life under normal vibration conditions could be too long so that the laboratory vibration tests at those levels would not be possible.

Accelerated testing provides the reduction in time and cost compared with the testing at normal conditions. In recent years, vibration fatigue accelerated testing methods have been continuously under development [1–5]. However, the vibration loadings are usually limited to sinusoidal or Gaussian random vibration, and the random vibration fatigue damage calculation is based on the assumption of Gaussian distribution. However, the dynamic environment shows non-Gaussianity in some practical applications, such as the ground vibration generated by wheeled vehicles travelling over irregular terrain. Fig. 1 plots the time history of a truck body vertical acceleration from road vibration measurements.

Because traditional Gaussian random vibration tests cannot accurately represent the non-Gaussian vibration environments with high-peak characteristics seen in the real-life use of some electronic products, the latest MIL-STD-810G test standard also requires test engineers to "ensure that test and analysis hardware and software are appropriate when non-Gaussian distributions are encountered" (refer to Method 525 on Page 514.6A-5 in literature [6]).

In this study, a link is to be established between the non-Gaussian vibration excitation and fatigue life, which will facilitate the design and statistical analysis of the accelerated vibration testing of electronic components.

2. Theoretical model

2.1. Model for Gaussian random vibration excitation

Through a series of derivations based on random vibration fatigue theory described in literature [7], the fatigue damage under Gaussian random vibration excitation can be calculated by:

$$D = k_1 T_G \left[\frac{G_a(f_1)}{\xi}\right]^{b/2} {f_1}^{(1-b/2)} \quad (1)$$

where D is cumulative fatigue damage, k_1 is a proportional constant, T_G is the duration of Gaussian random vibration excitation, $G_a(f_1)$ is the magnitude of the acceleration PSD of the random vibration excitation at f_1, f_1 is the first-order natural frequency of an electronic assembly,

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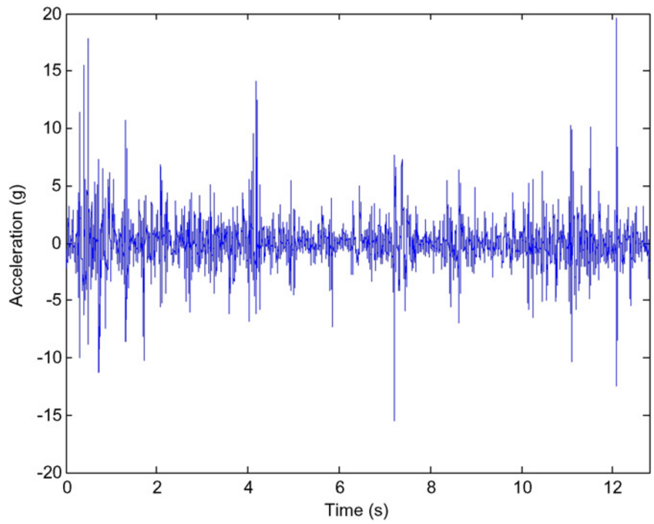

Fig. 1. Time history of a truck road vibration data.

$b$is the constant fatigue parameter that depends on the material of electronic components, and ξ denotes the equivalent damping ratio.

Fatigue failure is generally regarded to occur if $D = 1$. Then the vibration fatigue life T_G subject to Gaussian random vibration excitation can be obtained as:

$$T_G = \frac{{f_1}^{(b/2-1)}}{k_1}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2} \tag{2}$$

2.2. Model for non-Gaussian random vibration excitation

If the random stress response approximates a stationary non-Gaussian distribution, it is possible to add a non-Gaussian correction factor λ in Eq. (1) to describe the impact of the kurtosis value of the stress response on the cumulative vibration fatigue damage:

$$D = \lambda k_1 T\left[\frac{G_a(f_1)}{\xi}\right]^{b/2}{f_1}^{(1-b/2)} \tag{3}$$

According to fatigue theory, the higher the kurtosis value, the larger is the peak value, and the more extended the fatigue damage. So λ directly depends on the kurtosis value of stress responseK_s:

$$\lambda = 1 + \alpha(K_s - 3) \tag{4}$$

where α is a proportional coefficient.

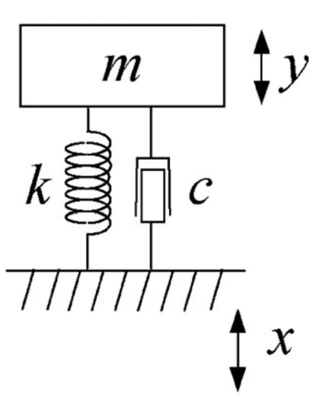

Fig. 2. Dynamic model of a base-excited SDOF system.

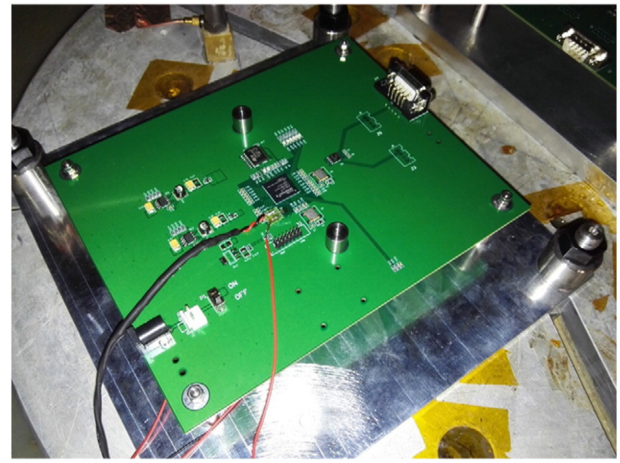

Fig. 3. Specimen of an electronic assembly mounted on a shaker.

Obviously, if the stress response is Gaussian, i.e. $K_s = 3$, $\lambda = 1$, Eq. (3) becomes Eq. (1). If the stress response is super-Gaussian, i.e. $K_s > 3$, $\lambda > 1$, the super-Gaussian stress response will accelerate the process of the fatigue damage.

The influence factors for K_s can be further analysed based on the random vibration theory. As the first-order mode of an electronic assembly plays a decisive role in the structural response, a single degree of freedom model under basic excitation is established for analysis as shown in Fig. 2.

The transfer function between the acceleration response y and basic acceleration excitation x can be derived as:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{cs + k}{ms^2 + cs + k} \tag{5}$$

Given

$$\omega_1 = \sqrt{\frac{k}{m}}, \quad \xi = \frac{c}{2\sqrt{mk}} \tag{6}$$

hence

$$H(s) = \frac{2\xi\omega_1 s + \omega_1^2}{s^2 + 2\xi\omega_1 s + \omega_1^2} \tag{7}$$

where$\omega_1 = 2\pi f_1$. f_1 denotes the first-order natural frequency, and ξ is the damping ratio. These two parameters characterise the structural

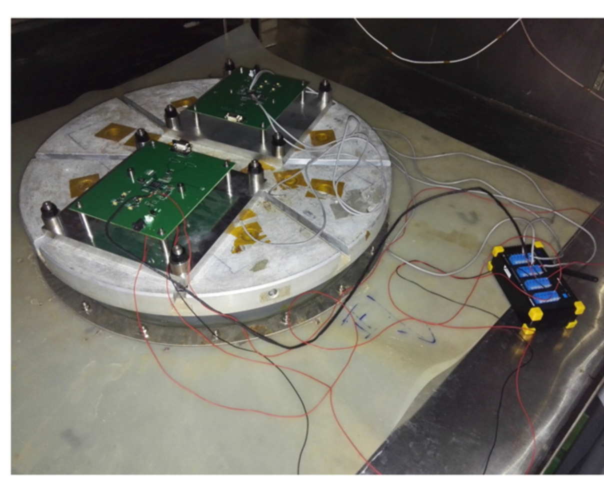

Fig. 4. Experimental setup for a vibration fatigue test.

Table 1
Test group A.

Parameters Number of profile	Lower frequency (Hz)	Upper frequency (Hz)	PSD bandwidth (Hz)	PSD magnitude (g^2/Hz)	G_{rms} (g)	Kurtosis
A1	10	160	150	0.20	5.48	3
A2	10	160	150	0.25	6.12	3
A3	10	160	150	0.30	6.71	3

dynamics of an electronic assembly and the pass-band width of an electronic assembly is also determined by these two parameters, namely

$$BW_H = 2\xi f_1 \tag{8}$$

Then the amplitude distribution of the structural response subject to the non-Gaussian excitation is discussed. The output of a stochastic process $x(t)$ through a linear system $H(f)$ can be represented in the time domain:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \tag{9}$$

Where $h(t)$ is the impulse response function of the system $H(f)$, the integral above can be formulated in terms of the sum-limit:

$$y(t) = \lim_{\substack{n\to\infty \\ \Delta\tau\to 0}} \sum_{k=1}^{n} x(\tau_k)h(t-\tau_k)\Delta\tau_k \tag{10}$$

where $x(\tau_k)$ is an input random variable, $\Delta\tau_k$ is the sampling interval. According to the central-limit theorem, the distribution of the sum of a large number of statistically independent random variables tends to be a Gaussian distribution. If the output of the random process at any time $y(t)$ is the sum of a large number of independent random variables, then $y(t)$ is close to a Gaussian distribution. Obviously it requires two conditions: one is that random variables must be independent of each other, and the other is that independent random variables are added to solve a sum.

Since the correlation time $\tau_x = \frac{1}{2BW_X}$ of the input random process is inversely proportional to the effective frequency bandwidth BW_X, τ_x will become very small when BW_X is very large. If τ_x is small enough to meet $\tau_x \ll \Delta\tau_k$, it can be considered that input samples of the random process are independent of each other. In other words, the random variables corresponding to τ_k as shown in $\sum_{k=1}^{n} x(\tau_k)h(t-\tau_k)\Delta\tau_k$ are independent of each other at any time t.

Now the second condition is considered. It is well known that when the random signal is applied to a narrowband system, the system can not immediately respond to the signal due to the system inertia. It requires a certain settling time t_s, and t_s is inversely proportional to the pass-band width BW_H of the system i.e. $t_s \propto 1/BW_H$. Thus, the smaller is BW_H, the greater is t_s. The longer is the response time of the signal, the longer becomes the cumulative time of the input random sampling (random variable). In this way, when the samples are independent of each other and the cumulative time is long enough i.e. $t_s \gg \Delta\tau_k$, $y(t)$ tends to be a Gaussian distribution. On the contrary, if a non-Gaussian random process $x(t)$ is applied to a linear system and the system pass-band BW_H is relatively wide, t_s is small. If $t_s \ll \Delta\tau_k$, the distortion of the input random process is very small after passing through the system. So the distribution of the output of the random process $y(t)$ will be close to the distribution of the original input $x(t)$, namely the non-Gaussian distribution.

In summary, if $\tau_x \ll \Delta\tau_k \ll t_s$ or simplified to $\tau_x \ll t_s$, the system output is close to the Gaussian distribution. Due to $\tau_x \propto 1/BW_X$ and $t_s \propto 1/BW_H$, $\tau_x \ll t_s$ means $BW_X \gg BW_H$. Thus these above conclusions can be expressed as: if the effective spectral bandwidth of the input random process for a linear system is much larger than the system pass-band width, the output amplitude distribution of the stochastic process will approximate a Gaussian distribution, no matter whether the input is Gaussian random process or not. In other words, when the effective spectral bandwidth of the input random process is close to or smaller than the system pass-band width, an input of non-Gaussian distribution will lead to an output of non-Gaussian distribution.

Based on the above analysis, the following equation can be used to describe the relationship among the kurtosis value K_s of the stress response, the kurtosis value K_a of the acceleration excitation, the bandwidth BW_a of the input acceleration excitation, and the pass-band width BW_H of an electronic assembly:

$$K_s = 3 + \beta\frac{BW_H}{BW_a}(K_a-3) \tag{11}$$

Combining the Eqs. (4) and (11) an expression for the non-Gaussian correction factor λ is obtained as:

$$\lambda = 1 + \alpha\beta\frac{BW_H}{BW_a}(K_a-3) \tag{12}$$

From Eq. (12), it can be seen that the kurtosis and bandwidth of a non-Gaussian random vibration excitation have a strong influence on the non-Gaussian feature of the structural stress response and further significantly affect the vibration fatigue life.

Inserting Eq. (12) into Eq. (3) and assigning $D=1$, the vibration fatigue life T_{NG} under non-Gaussian random vibration excitation is presented as:

$$T_{NG} = \frac{{f_1}^{(b/2-1)}}{k_1\left[1 + \varepsilon\frac{BW_H}{BW_a}(K_a-3)\right]}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2} \tag{13}$$

where $\varepsilon = \alpha\beta$ is the synthesized non-Gaussian correction factor.

With the above mathematical model, it is possible to predict the vibration fatigue life of electronic components under non-Gaussian random vibration excitations, which will be explained in the following section.

2.3. Method of determining the unknown parameters

From Eq. (13) it can be seen that once the particular electronic components and assemblies are determined, the three parameters(f_1, ξ and BW_H) will be determined correspondingly and can be regarded as known parameters. Further, once the vibration excitation condition is determined, $G_a(f_1)$ and $BW_a$are also determined and can be also regarded as known parameters. Thus, there are only three unknown parameters(i.e. b, k_1 and ε) to be determined for the fatigue life prediction

Table 2
Test group B.

Parameters Number of profile	Lower frequency (Hz)	Upper frequency (Hz)	PSD bandwidth (Hz)	PSD magnitude (g^2/Hz)	G_{rms} (g)	Kurtosis
B1	60	110	50	0.25	3.54	5
B2	60	110	50	0.25	3.54	7

model described by Eq. (13). The method to determine these three parameters is discussed below.

Firstly we can solve the parameters b and k_1 based on the results of Gaussian vibration fatigue test.

According to Eq. (2), the vibration fatigue lives under two different Gaussian random acceleration excitations which are achieved respectively as:

$$T_{G1} = \frac{f_1^{(b/2-1)}}{k_1}\left[\frac{\xi}{G_{a1}(f_1)}\right]^{b/2} \tag{14}$$

$$T_{G2} = \frac{f_1^{(b/2-1)}}{k_1}\left[\frac{\xi}{G_{a2}(f_1)}\right]^{b/2} \tag{15}$$

Thus

$$\frac{T_{G1}}{T_{G2}} = \left[\frac{G_{a2}(f_1)}{G_{a1}(f_1)}\right]^{b/2} \tag{16}$$

Taking the logarithm on both sides of Eq. (16)

$$\ln\frac{T_{G1}}{T_{G2}} = \frac{b}{2}\ln\left[\frac{G_{a2}(f_1)}{G_{a1}(f_1)}\right] \tag{17}$$

is obtained.

Given $Y_1 = \ln\frac{T_{G1}}{T_{G2}}$ and $X_1 = \ln\frac{G_{a2}(f_1)}{G_{a1}(f_1)}$, Eq. (17) is simplified to

$$Y_1 = \frac{b}{2}X_1 \tag{18}$$

Therefore, we can carry out a series of Gaussian vibration fatigue accelerated tests by only changing the power spectral density $G_a(f_1)$ of the Gaussian random vibration excitation. Based on the above test results, a series of values of the group (X_1, Y_1) can be obtained and fitted to a straight line. Then the value of parameter b can be estimated from the slope of the line.

Then the following discussion is about how to solve the parameter k_1 based on the results of Gaussian vibration fatigue accelerated testing.

Eq. (2) can be also transformed to:

$$f_1^{(b/2-1)}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2} = k_1 T_G \tag{19}$$

Given $Y_2 = f_1^{(b/2-1)}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2}$ and $X_2 = T_G$, Eq. (19) is simplified to

$$Y_2 = k_1 X_2 \tag{20}$$

Similarly, a series of values of the group (X_2, Y_2) can be obtained based on the above test results and fitted to a straight line. Then the value of parameter k_1 can be estimated from the slope of the line.

Finally, we study how to determine the parameter ε based on the results of non-Gaussian vibration fatigue accelerated testing.

Eq. (13) can be transformed to:

$$\frac{f_1^{(b/2-1)}}{k_1 T_{NG}}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2} - 1 = \varepsilon\frac{BW_H}{BW_a}(K_a - 3) \tag{21}$$

Given $Y_3 = \frac{f_1^{(b/2-1)}}{k_1 T_{NG}}\left[\frac{\xi}{G_a(f_1)}\right]^{b/2} - 1$ and $X_3 = \frac{BW_H}{BW_a}(K_a - 3)$, Eq. (21) is simplified to:

$$Y_3 = \varepsilon X_3 \tag{22}$$

Therefore, we can carry out a series of non-Gaussian vibration fatigue accelerated testing by changing the kurtosis value K_a or the bandwidth BW_a of the non-Gaussian random vibration excitation. Based on the above test results, a series of values of the group (X_3, Y_3) can be obtained and fitted to a straight line. Then the value of parameter ε can be estimated from the slope of the line.

Table 3
Test group C.

Parameters Number of profile	Lower frequency (Hz)	Upper frequency (Hz)	PSD bandwidth (Hz)	PSD magnitude (g^2/Hz)	G_{rms} (g)	Kurtosis
C1	10	160	150	0.15	4.74	3
C2	75	95	20	0.25	2.24	5
C3	75	95	20	0.25	2.24	7

3. Experimental verification

3.1. Experimental setup

Fig. 3 shows the specimen of an electronic assembly mounted on a shaker. The specimen is a specially designed PCB with Ball Grid Array (BGA) packages attached, and its status can be indicated by a luminous diode and a buzzer.

Fig. 4 illustrates the experimental setup for accelerated random vibration fatigue testing. To save test time, two specimens were installed simultaneously for testing.

The closed-loop vibration fatigue testing system consists of a shaker, a power amplifier, a vibration controller (VT-9008, Econ Corporation), and two accelerometers (one for vibration control, the other for monitoring the response of an electronic assembly). In addition to complete the traditional sine and Gaussian random vibration test, the VT-9008 vibration controller is capable of generating a non-Gaussian random vibration signal with the specified power spectrum density and kurtosis value, which can be used to study the non-Gaussian random vibration fatigue.

3.2. Experimental procedure

Before the random vibration fatigue tests, the dynamic transfer characteristics of the specimens were tested by a sweep sine test. Then we can determine that the first-order natural frequency f_1 is 85 Hz, damping ratio ξ is 0.02, and the pass-band width of the specimen BW_H is 3.4 Hz.

According to the above method of solving the unknown three parameters, two groups of random vibration fatigue tests were designed and the corresponding vibration excitation profiles are listed in Tables 1–2 (all PSD shapes in the tables are flat for simplicity).

Group A comprises Gaussian random vibration fatigue accelerated tests, and its results will be used to find the parameters b and k_1.

Group B comprises non-Gaussian random vibration fatigue accelerated tests by changing the kurtosis value K_a, and its results will be used to find the parameters ε.

Table 4
Test results.

Test group	Number of profile	Number of specimen	T (minute)	T_e (minute)
A	A1	4	166; 192; 183; 199	185
	A2	4	105; 96; 123; 116	110
	A3	4	87; 72; 65; 76	75
B	B1	4	99; 79; 86; 96	90
	B2	4	75; 67; 79; 59	70
C	C1	4	371; 377; 402; 390	385
	C2	4	68; 78; 85; 73	76
	C3	4	62; 53; 47; 58	55

where T is the actual experimented time to failure; T_e is its mean.

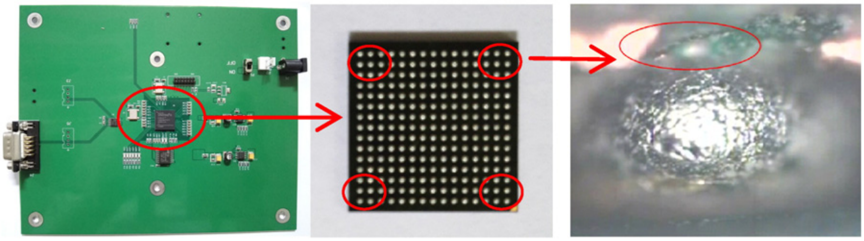

Fig. 5. Micrograph of a failed BGA solder ball with a fatigue crack.

After determining the unknown parameters, group C is carried out to verify the fatigue life prediction model. (See Table 3.)

3.3. Experimental results

Detailed test results are shown in Table 4.

The BGA component is specifically installed at the center of the PCB where the maximum amplitude of vibration is located. Thus all the test specimens have the same vibration fatigue failure modes at the solder joints of the BGA components. Moreover, the positions of the failed solder balls are all distributed at four corners of the BGA components, marked by red circles as shown in Fig. 5. It indicates that the physical failure modes of test specimens and their positions correspond to each other.

According to the test results of Group A, the value of parameter b is estimated as $b = 4.4381$, and the value of parameter k_1 is estimated as $k_1 = 0.0074$.

According to the test results of Group B, the value of parameter ε is estimated as $\varepsilon = 2.098$.

Then the vibration fatigue lives under different vibration excitation profiles in Group C are predicted by the fatigue life prediction model described by Eq. (13). The predicted fatigue lives for C1, C2 and C3 are 348 min, 67 min and 47 min respectively, which are compared with the corresponding experimental results of Group C in Table 4. The relative prediction errors for C1, C2 and C3 are 9.61%, 11.84% and 14.55% respectively, which generally meets the requirement of engineering applications.

4. Conclusions

In this paper, a novel fatigue life prediction model for electronic components under non-Gaussian random vibration excitations is proposed based on random vibration and fatigue theory. The main conclusions are listed as follows:

(1) The proposed model comprehensively associates the vibration fatigue life of electronic components, the characteristics of non-Gaussian random vibration excitations (such as the root mean square, power spectral density, spectral bandwidth and kurtosis value) and the dynamic transfer characteristics of an electronic assembly (such as the natural frequency and damping ratio) together. This will facilitate the quantitative design and statistical analysis of non-Gaussian random vibration fatigue accelerated testing.

(2) A practical method was also presented for determining the unknown parameters in the above model, and only a few vibration fatigue accelerated tests are needed to be carried out in the laboratory. Then it is easy to predict the vibration fatigue life of electronic components under specified Gaussian or non-Gaussian random vibration environments such as given in Fig. 1. The experimental results of a series of random vibration fatigue accelerated tests show the validity and practicability of the fatigue life prediction model.

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